Effective chiral restoration in the hadronic spectrum and QCD

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Effective chiral restoration in the hadronic spectrum has been conjectured as an explanation of multiplets of nearly degenerate seen in highly excited hadrons. The conjecture depends on the states being insensitive to the dynamics of spontaneous chiral symmetry breaking. A key question is whether this concept is well defined in QCD. This paper shows that it is by means of an explicit formal construction. This construction allows one to characterize this sensitivity for any observable calculable in QCD in Euclidean space via a functional integral. The construction depends on a generalization of the Banks-Casher theorem. It exploits the fact that *all* dynamics sensitive to spontaneous chiral symmetry breaking observables in correlation functions arise from fermion modes of zero virtuality (in the infinite volume limit), while such modes make *no* contribution to any of the dynamics which preserves chiral symmetry. In principle this construction can be implemented in lattice QCD. The prospect of a practical lattice implementation yielding a direct numerical test of the concept of effective chiral restoration is discussed.

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I. INTRODUCTION

A striking feature of the hadron spectrum has a number of cases in which states of the same spin and opposite parity are nearly degenerate. One suggestion is that this phenomenon is the result of "effective restoration" of chiral symmetry in the spectrum[1, 2, 3, 4, 5]. The notion that chiral multiplets may be relevant to hadron spectrum in the context of simple phenomenological models was discussed in ref. [6]. If this conjecture is correct the spectrum should consist of approximate chiral multiplets rather than simply have parity doublets[2]. It has been argued that there are indications in the hadron spectrum for such patterns. The basic theoretic idea underlying this conjecture is that if the properties of the high-lying hadrons are largely insensitive to the dynamics of spontaneous chiral symmetry breaking, then to good approximation the states will fall into chiral multiplets. The logic is simple: although the system is in the Nambu-Goldstone phase, the particular states are insensitive to this fact and the spectra act to good approximation as if they were in the Wigner-Weyl phase. There is no theoretical reason why this is excluded, and a simple calculable model illustrates how the phenomenon could come about[7]. Moreover, there are some general arguments as to why this should not be regarded as implausible[2, 3, 4, 5, 8]. At the same time, it should be noted that such an interpretation remains controversial[9, 10] and the empirical evidence from the spectrum from the spectrum is probably better characterized as being suggestive rather than compelling. While there is additional support for the phenomenon based on the small size of the couplings of excited hadrons to pions[11], the overall phenomenological evidence is not conclusive.

Given this somewhat unsatisfactory situation, it is important to ask whether there is any theoretical method to determine if effective chiral restoration occurs to good approximation in the spectrum of QCD. There are actually two related issues here. The first is one of principle: Can one formulate a calculation in QCD which, if done, determines whether effective chiral restoration occurs to good approximation? As we will demonstrate, the answer to this question is "yes". The second issue is more practical: namely, is there a viable method to actually implement such a calculation? As will be discussed below, the answer to this question depends at minimum on the existence of reliable lattice calculations for the masses of excited hadronic resonances. The problem of determining whether excited resonant states exhibit effective chiral restoration from the lattice may require substantially more computational resources than simply determining the masses as it requires enough numerical power to approach multiple limits in a particular order. Nevertheless, there is a real prospect of using lattice data (at some future date) to determine whether or not effective chiral restoration is, in fact, responsible for the degeneracies seen in the spectrum for states of opposite parity.

The crux of the theoretical issue is that the effective chiral restoration depends on the masses of the excited hadrons be largely insensitive to the physics of spontaneous chiral symmetry breaking. To proceed further it is essential to formulate more precisely what this means. In the context of simple mean field models such as the one considered in ref. [7], it is easy to answer this—there is a single chiral order parameter in the models and its size is controlled directly from an external parameter. Hence, in the model one can directly test how the mass varies with the order parameter. One would like to do the same thing with QCD. However, there are problems with this. In the first place we can not independently alter the chiral condensate. Of course, one can add something to the theory (say, a chemical potential) which alters the

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chiral condensate and ask how much the hadron's mass changes. The difficulty with this is that we do not know how much of the change in the hadron's mass is due to the changing condensate and how much is due to other effects associated with adding something new to the theory. Moreover, the chiral condensate is not the only chiral order parameter.

Thus, to proceed we need some algorithm by which we can turn off the effects of spontaneous chiral symmetry breaking on observables in QCD without altering the dynamics not associated with chiral symmetry breaking. Given such an algorithm one can calculate the hadron mass with the effects of spontaneous chiral symmetry breaking included and excluded, and directly compare the differences.

The first thing to note is that all physical processes involving hadrons are ultimately describable in terms of correlations functions of gauge invariant currents. The correlation functions may be regarded as the response of the system given certain types of probes which are determined by the currents. Thus, if one can find two sets of currents, one of which probes the system in the conventional way and a second set which are identical in all respects except that they do not probe the chiral symmetry breaking aspects of the system, in effect one would be able to turn off the dynamics of chiral symmetry breaking. In doing so it is important to note that the correlation functions are describable in terms of dispersion relations with the information contained in spectral functions. The key issue is whether the spectral functions in the regions of interest are dominated by the dynamics which preserve chiral symmetry.

The goal is to isolate the dynamics of chiral symmetry breaking inside the calculation. While it is not clear how to formulate such an algorithm in Minkowski space, there is a rather straightforward way to do so in Euclidean space. The key hint in constructing such currents is an old observation of Banks and Casher[12] that the chiral condensate in a Eulclidean formulation is directly proportional to the density of states of the Dirac operator at zero virtuality:

$$\langle \overline{q} q \rangle = -\pi \rho(0) \ . \tag{1}$$

The Banks-Casher relation applies only to the chiral condensate. However, the result applies far more generally. In the appropriate chiral and infinite volume limits (with $V \to \infty$ taken first) *all* effects of chiral symmetry breaking on *any* Eulcidean space correlation function are directly attributable to the contribution of fermion modes associated with the external currents that have zero virtuality. Moreover, these modes that contribute *only* to dynamics which break chiral symmetry. By comparing correlations functions calculated with zero modes included in the external currents with analogous calculations excluding them, one has a direct measure of the role of chiral symmetry breaking on the Euclidean correlation function.

As will be discussed in detail below, this is a highly nontrivial numerical problem in practice. However, some insight into this behavior has already been extracted from lattice simulations by Degrand[13], who showed that the near zero modes contribute strongly to the correlation functions at long times but had a very small contribution to the short-distance behavior. While calculations of this sort are not suitable for demonstrating effective chiral restoration in the spectrum they provide a useful consistency check. The results of ref. [13] are consistent with the notion of effective chiral restoration.

For simplicity this paper will only consider non-strange systems, so the relevant chiral symmetry is $SU(2)_L \times SU(2)_R$ and will focus on the chiral limit of zero quark mass. The paper is organized as follows: In the next section the issue of how to obtain the masses of resonant states from correlation functions is addressed briefly. The subsequent section demonstrates that in Euclidean correlation functions the zero virtuality modes contribute to effects which spontaneously break chiral symmetry and only to such effect. Next an explicit construction is made for correlation functions removing the effects of dynamical symmetry breaking. Following this is a section discussing the role of the $U(1)$ axial anomaly in the preceding analysis. Finally, there is a section which addresses the prospects for implementing it on the lattice.

II. HADRONIC RESONANCES FROM EUCLIDEAN SPACE CORRELATION FUNCTIONS

Dispersion relations give both Euclidean space and Minkowski space correlation functions from the same spectral function. Thus, if one can deduce the spectral function from a Euclidean space calculation, then one knows the Minkowski space amplitudes. Since all observable properties of hadronic states and their interactions can be extracted from Minkowski correlation functions, all one needs is an algorithm to extract the appropriate spectral function.

As a simple example, note that prominent resonances may appear directly in the spectral function for a two-point correlation function. Ignoring inessential complications of spin and flavor, the Euclidean space two-point correlator for a local gauge invariant current J is given by:

$$\Pi(x) \equiv \langle J^\dagger(x) J(0) \rangle = \int ds \, \rho(s) G(x;s) \tag{2}$$

where $G(x;s)$ is the scalar propagator for a point particle of mass $\sqrt{s}$:

$$G(x;s) = \int \frac{d^4Q}{(2\pi)^4} \, \frac{e^{i(\vec{Q}\cdot x + Q_0 t)}}{Q^2 + s} \ .$$

A prominent resonance corresponds to a region of considerable spectral strength over a relatively small area. A caricature of such a spectral function is shown in fig. 1

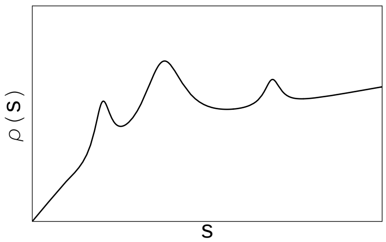

FIG. 1: Caricature of a spectral function containing three clear resonant structures

As was noted in the introduction, the prescription developed in this paper here to isolate the effects of spontaneous chiral symmetry breaking only works in Euclidean space. On one hand, this is not a serious drawback. After all, the only practical method to calculate hadronic quantities from QCD in an *ab initio* way is via lattice QCD in Euclidean space. On the other hand, extracting information about excited states from Euclidean space calculations is intrinsically hard—at least numerically—and extracting information about resonances is harder. Fortunately, for the purpose of establishing the theoretical result that the notion of effective chiral restoration is well defined, it is sufficient to show that properties of hadronic resonances can be computed from Euclidean space correlation functions as a matter of principle. Thus, the central issue of principle is whether the spectral functions can be extracted given knowledge of the Euclidean space correlation functions. Since the correlation function is an integral transform of the spectral function, the problem amounts to finding the inverse of the transform. The problem is easily converted to a standard one of the inverse Laplace transform by focusing on the spatial integral of the correlation function at fixed time:

$$\Pi(t) \equiv \int d^3x\, \Pi(\vec{x},t) \; = \; \int ds\, \frac{\rho(s)}{2\sqrt{s}}\, e^{-\sqrt{s}t} \; . \qquad (3)$$

Unfortunately, extracting inverse Laplace transform is a notoriously difficult problem—indeed, it is known to be numerically unstable. However, given sufficiently accurate computations for spectral functions there are methods which produce spectra that can be made to closely approximate the correct one to as much accuracy as one wishes. These techniques suffer from an important practical defect—namely, that the numerical difficulty of computing at fixed level of accuracy grows exponentially as one goes up in the spectrum and the difficulty of computing at an increased level accuracy at a fixed s also grows exponentially with accuracy. However for the issue of principle it is sufficient to know that it is computable.

A typical method is straightforward: one characterizes the spectral function by some functional form with a fixed number of parameters, N. One calculates $\Pi(t)$ at N distinct values of t chosen according to a prefixed scheme. One then fits the parameters to ensure the correct value of the correlation function at theses points. The spectral function is increasingly well described as $N \to \infty$.

A particularly useful way to parameterize the spectral function is via $N/2$ poles, each of variable strength. Note that for any finite value of N this will be a very poor approximation of the spectral function in a point-by-point sense. However, this is not relevant; for a sufficiently low part of the spectrum it will accurately reproduce the integrals of spectral strength over regions large compared with the typical level spacing. One can replace the spectral function constructed by the poles by a histogram. As N gets large, such histograms will approximate the spectral function with increasing accuracy. If N is large enough there will be many poles over the characteristic scale of the resonances and one can then resolve the resonant structure. Note that exact calculations of correlation functions on systems of finite spatial extent *always* yield discrete poles. As the volume becomes large the poles become dense. It is relatively easy to extract the position and residue of the lowest pole but it becomes exponentially harder to extract poles as one goes up in energy. Nevertheless with sufficient numerical precision one can extract as many poles needed to probe the spectral function in a given region.

Thus, given sufficiently accurate Euclidean correlation functions one can reconstruct the spectral functions well enough to resolve resonant structures with as much accuracy as we wish.

III. SPONTANEOUS CHIRAL SYMMETRY BREAKING AND MODES OF ZERO VIRTUALITY

Euclidean space correlation functions in QCD for currents may be written as a functional integral over gluonic fields. The structure may be quite complicated and may have many terms. However, all terms are contractions (in color, flavor and Dirac index) of structures of the following form:

$$\frac{\int D[A]\, E^{-S_{YM}} \prod_f Det\,(\not{D} + m_f)\, f[A] \prod_{k=1}^{k_{\max}} G_A}{\int D[A]\, E^{-S_{YM}} \prod_f Det\,(\not{D} + m_f)} \qquad (4)$$

where A is a gluon background configuration, S_{YM} is the Yang-Mills action for the configuration, f indicates flavor, $G_A = (\not{D} + m)$ is the quark propagator in the presence of the gluon background field A, k_{max} indicates the total number of quark propagators in the term and $f(A)$ is associated with any explicit gluon fields in the current. We are interested in chiral symmetry and hence in the $m_f \to 0$ limit.

Chiral invariance is the decoupling of the left-handed and right-handed quark fields. The only term in the QCD Lagrangian which couples left-handed and right-handed quarks are the mass terms. Chiral symmetry breaking is the coupling of left-handed and right-handed quarks in the limit of $m_q \to 0$. The observable chiral symmetry breaking effects in correlation functions necessarily arise from the coupling of left-handed fields and right-handed fields in the external currents. This in turn means that if we can construct currents which do not couple left-handed and right-handed quarks, we will suppress all effects of chiral symmetry breaking.

In terms of Eq. (4) the only way left-handed and right-handed quark fields couple is through the propagators G_A. One can always decompose the propagator into a chiral conserving part (which couples left-handed quarks to left-handed quarks and right-handed quarks to right-handed quarks) and a chiral breaking part which couple left-handed quarks to right-handed quarks.

$$G_A = G_A^c + G_A^b \tag{5}$$

$$G_A^c = \frac{(1+\gamma_5)G_A(1-\gamma_5) \, + \, (1-\gamma_5)G_A(1+\gamma_5)}{4}$$

$$G_A^b = \frac{(1+\gamma_5)G_A(1+\gamma_5) \, + \, (1-\gamma_5)G_A(1-\gamma_5)}{4} \, .$$

Before proceeding it is useful to illustrate the general results with a specific example. Consider the vector and axial-vector isovector currents (associated with the ρ and A_1 channels):

$$J^{\mu\, a} \equiv \overline{q}\gamma^\mu \tau_a q \quad J_A^{\mu\, a} \equiv \overline{q}\gamma_5\gamma^\mu \tau_a q \tag{6}$$

These two currents together form a $(3,0)+(0,3)$ chiral-parity multiplet; they transform into each other under axial rotations. It is straightforward to see that

$$\begin{aligned} \Pi^{\mu\nu}(x) &\equiv \langle J^{\mu\, a}(x) J^{\nu\, a}(0)\rangle = 4\langle\langle \gamma^\mu G^c(x,0)\gamma^\nu G^b(0,x)\rangle\rangle + 4\langle\langle \gamma^\mu G^b(x,0)\gamma^\nu G^c(0,x)\rangle\rangle \\ \Pi_A^{\mu\nu}(x) &\equiv -\langle J_A^{\mu\, a}(x) J_A^{\nu\, a}(0)\rangle = 4\langle\langle \gamma^\mu G^c(x,0)\gamma^\nu G^c(0,x)\rangle\rangle - 4\langle\langle \gamma^\mu G^c(x,0)\gamma^\nu G^c(0,x)\rangle\rangle \end{aligned} \tag{7}$$

where the $\langle\langle\rangle\rangle$ notation is shorthand for averaging over gluonic configurations weighted by $e^{-S_{YM}}\prod_f Det\,(\not{D}+m_f)$. The critical issue illustrated here is that the *only* thing separating these correlation functions is the contributions from the chiral breaking parts of the correlators. This property is generic.

We ultimately wish to derive a Banks-Casher[12] type relation in which we show that in the appropriate limit all of the chiral symmetry breaking effects come from states with zero virtuality. Thus it is instructive to write these propagators in terms of eigenmodes of the Dirac operator. To do this we first put the system into a finite space-time box by imposing appropriate boundary conditions. This causes the eigenmodes to be discrete. We will take the infinite volume limit at a later stage. The propagator can then be expressed as

$$G_A(x,x') = \sum_j \frac{\psi_j(x)\psi_j^\dagger(x')}{i\lambda_j + m} \tag{8}$$

where $\psi_j(x)$ is an eigenfunction of the Dirac operator $\not{D}$ with eigenvalue $i\lambda_j$ (color, flavor and dirac indices have been suppressed). Combining Eq. (8) with Eq. (5) one can write the chiral-conserving and chiral-breaking propagators in terms of density matrices composed of the modes:

$$\begin{aligned} G_A^c &= \int_{-\infty}^{\infty} d\lambda\, \frac{\rho_A^c(x,x':\lambda)}{i\lambda+m} \quad G_A^b = \int_{-\infty}^{\infty} d\lambda\, \frac{\rho_A^b(x,x';\lambda)}{i\lambda+m} \\ \rho_A^c(x,x';\lambda) &= \sum_j \delta(\lambda-\lambda_j)\, \frac{(1+\gamma_5)\psi_j(x)\psi_j^\dagger(x')(1-\gamma_5) \, + \, (1-\gamma_5)\psi_j(x)\psi_j^\dagger(x')(1+\gamma_5)}{4} \\ \rho_A^b(x,x':\lambda) &= \sum_j \delta(\lambda-\lambda_j)\, \frac{(1+\gamma_5)\psi_j(x)\psi_j^\dagger(x')(1+\gamma_5) \, + \, (1-\gamma_5)\psi_j(x)\psi_j^\dagger(x')(1-\gamma_5)}{4} \, . \end{aligned} \tag{9}$$

It is straightforward to derive some general properties of the propagator using the spectral decomposition. Since $\not{D}$ anti-commutes with γ_5, every eigenfucntion ψ_j with a non-zero eigenvalue $i\lambda_j$ has a partner $\gamma_5\psi_j$ which is also an eigenvector and has eignevalue $-i\lambda_j$; thus non-zero modes come in pairs. Zero modes are special: one can have unpaired zero modes provided $\gamma_5\psi = \pm\psi$. Thus unpaired zero modes always have fixed chirality either left or right. This in turn means that the unpaired zero modes do not contribute to ρ^c but can contribute to ρ^b. Using this fact plus paired structure of the eigemodes it is easy to see that

$$\begin{aligned} \rho^c_A(x,x';\lambda)(x,x') &= -\rho^c(-\lambda)(x,x') \\ \rho^b_A(x,x';\lambda) &= \rho^b(x,x';-\lambda)\ . \end{aligned} \tag{10}$$

Thus the chiral conserving and chiral breaking parts of the spectral function can be written as

$$\begin{aligned} G^c_A &= \int_{-\infty}^{\infty} d\lambda\ \rho^c_A(x,x';\lambda)\,\frac{-i\lambda}{\lambda^2+m^2} \\ G^b_A &= \int_{-\infty}^{\infty} d\lambda\ \rho^b_A(x,x';\lambda)\,\frac{m}{\lambda^2+m^2} \end{aligned} \tag{11}$$

We are interested in the chiral limit of $m \to 0$. Naively it appears that G^c_A will be nonzero in this limit while G^b_A automatically vanishes. Of course, this is not quite right. In order for spontaneous symmetry breaking to occur one must take the limit $V \to \infty$ prior to the $m \to 0$ limit. On can proceed rather simply in a formal manner. First take the $V \to \infty$ limit. This renders the ρ^b and ρ^c as continuous functions of λ rather than as a discrete sum over delta functions in Eqs. (10). One can then take the $m \to 0$ limit using the fact that $\lim_{m\to 0} \frac{m}{\lambda^2+m^2} = \pi\delta(\lambda)$:

$$G^c_A(x,x') = \int_0^{\infty} d\lambda\ \frac{-2i\rho^c_A(x,x';\lambda)}{\lambda} \tag{12}$$

$$G^b_A(x,x') = \pi\rho^b_A(x,x';0)\ . \tag{13}$$

Equation (13) may be regarded as a generalized Banks-Casher formula. The key point here for our purposes is that $G^b_A(x,x';\lambda)$ *only* gets contributions from the neighborhood of zero virtuality in the infinite volume and chiral limits while $G^c_A(x,x';\lambda)$ gets no contributions from modes approaching zero virtuality.

This generalized Banks-Casher formula can be combined with the fact that all chiral symmetry breaking effects in Euclidean correlation functions ultimately are traced to the G^b_A's in the functional integrals. This allows one to deduce the general result that *all* chiral symmetry breaking effects to zero mode contributions. Thus, for example if one were to consider the case of the vector and axial correlation functions from Eq. (7) one sees that

$$\Pi^{\mu\nu}(x) - \Pi^{\mu\nu}_A(x) = 8\pi^2\langle\langle\gamma^\mu\rho^b_A(x,0;0)\gamma^\nu\rho^b_A(0,x;0)\rangle\rangle\ . \tag{14}$$

The difference of the vector and axial correlators is a direct measure of an effect of spontaneous chiral symmetry breaking and it is completely determined by the density matrices at zero virtuality.

IV. EXCLUDING MODES OF NEAR ZERO VIRTUALITY

As noted above, all effects of chiral symmetry breaking in Euclidean correlation functions are due to contributions from of modes which go to zero virtuality in the infinite volume limit in quark propagators connected to external currents. Thus, by excluding all of these modes—and only these modes—from a calculation of a correlation function one is removing the effects of chiral symmetry breaking while doing no other violence to the dynamics. This section proposes a simple explicit construction which allows this to be done in a simple and transparent way while preserving all other features of QCD.

The key point is that the dependence is only for quark propagators connected to external currents. Thus if one were able to construct external currents which couple to the modes with non-zero virtuality in the standard way while not coupling to modes in the neighborhood of zero one would have constructed a direct way to probe the sensitivity of the correlation function to chiral symmetry breaking.

The basic tool in constructing such currents is a non-local quark field operator defined as

$$q^\spadesuit(x;\lambda_0) \equiv \theta\left(-\not{D}^2 - \lambda_0^2\right) q(x) \tag{15}$$

where θ is the standard step-function and the spade notation indicates that the contribution from all modes whose virtuality has a magnitude less than λ_0 have been "dug out". Ultimately the limit of $\lambda_0 \to 0$ will be taken so that only the immediate neighborhood of zero virtuality will be excluded. One disadvantage of this field operator is that it is non-local. However, this does not prevent it being used to compute correlation functions. A second disadvantage is that there is no obvious way to interpret the field in Minkowski space. However, this should not be viewed too negatively—the entire issue of zero modes is only relevant in Euclidean space. More to the point the calculation of correlation functions based on currents constructed from these operators are well defined in Euclidean space.

The transformation properties of $q^\spadesuit(x;\lambda_0)$ under gauge transformations is simple: they transform the same way as ordinary quark fields;

$$q^\spadesuit(x;\lambda_0) \to U(x)q^\spadesuit(x;\lambda_0)\ . \tag{16}$$

This is easy to see. Recall that $\not{D} \to U(x)\not{D}U^\dagger(x)$ which implies that any function of $\not{D}$ transforms according to $f(\not{D}) \to U(x)f(\not{D})U^\dagger(x)$. Thus

$$\theta\left(-\not{D}^2 - \lambda_0^2\right) q(x) \to U(x)\theta\left(-\not{D}^2 - \lambda_0^2\right) U^\dagger(x)U(x)q(x) \tag{17}$$

from which Eq. (16) immediately follows.

Since $q^\spadesuit(x;\lambda_0)$ transforms in the same way as q under gauge transformations, for every gauge-invariant current constructed from ordinary quark fields is a corresponding gauge invariant current constructed from the $q^\spadesuit$. So, for example, one can define a vector-isovector current out of these fields:

$$J^{\spadesuit\,\mu\,a}(x;\lambda_0) \equiv \overline{q}^\spadesuit(x;\lambda_0)\,\gamma^\mu\tau_a\,q^\spadesuit(x;\lambda_0) \tag{18}$$

Similarly one can construct correlation functions using these spaded currents . We will denote these with a spade.

Generically these spaded-correlators can depend on the volume, the quark mass and λ_0. We wish to study the infinite volume, zero mass and $\lambda_0 \to 0$ limits of these. Clearly the ordering of the limits matters. We will always take the infinite volume limit first. If we take the $\lambda_0 \to 0$ limit first, one reproduces the correlator for the standard current in the chiral limit

$$\lim_{m\to 0}\lim_{\lambda_0\to 0}\Pi^{\spadesuit}(x;m,\lambda_0) = \lim_{m\to 0}\Pi(x,m) \equiv \Pi(x) \qquad (19)$$

where Π indicates a generic correlation function in Euclidean space. In contrast, if one first takes the $m \to 0$ limit and then $\lambda_0 \to 0$ limit,

$$\lim_{\lambda_0\to 0}\lim_{m\to 0}\Pi^{\spadesuit}(x;m,\lambda_0) \equiv \Pi^{\spadesuit}(x) \;, \qquad (20)$$

one has completely removed all effects of chiral symmetry breaking while maintaining all chiral conserving effects.

Finally we write correlators as integral transforms of spectral functions. For the case of two-point functions it is given by

$$\begin{aligned}\Pi(x) &\equiv \langle J^{\dagger}(x)J(0)\rangle = \int ds\,\rho(s)G(x;s)\\ \Pi^{\spadesuit}(x) &\equiv \langle J^{\dagger}(x)J(0)\rangle = \int ds\,\rho^{\spadesuit}(s)G(x;s)\;. \qquad (21)\end{aligned}$$

As noted earlier this step is highly nontrivial numerically due to the difficulty in inverting the transform, but with sufficient precise correlation functions it can be done. Before proceeding it is worth noting here that the non-local nature of the fields in the spaded currents means that usual positivity bounds on the spectral function need not apply.

We have reached the final stage of the construction. In principle, provided we have accurate Euclidean space correlators, we now know how to compute the spectral functions with the chiral symmetry breaking effects included or excluded. To the extent that the spectral functions are dominated by the chiral conserving dynamics with s in a region interest, the system is in the regime of effective chiral restoration. One can set the precise criteria for identifying effective chiral restoration based on how dominant one insights the chiral conserving parts must be. In regions where resonance with fixed quantum numbers are not close together compared to typical hadronic scales a reasonable criterion is $\rho(s) >> \rho^{\spadesuit}(s)$ and $|\partial_s\rho(s)| >> |\partial_s\rho^{\spadesuit}(s)|$.

One might worry that the scheme alters the dynamics too violently to be useful. A particular concern is that the removal of the zero modes might somehow undo the dynamics of confinement and kill the resonant structures in the spaded spectral functions. However, this cannot be the case. This can be seen clearly by looking at the vector and axial vector spectral functions. As seen in this section, the sum of the two spectral functions is identical to the sum of the spaded spectral functions while the differences are zero. If a resonance exist in either channel it will appear in the sum of the two and be present in the spaded spectral function.

Thus, as a matter of principle it is clear how to test the extent to which effective chiral restoration occurs in QCD. However, the real practical question of interest is not whether a regime of approximate effective chiral restoration occurs in the spectral functions. On very general grounds one expect this to happen at sufficiently large large s [2, 8]. The central question is whether this regime occurs at low enough s so that individual resonances are discernible. Direct calculations of $\rho(s)$ and $\rho^{\spadesuit}(s)$ from QCD would enable this question to be answered.

V. EFFECTS OF $U(1)_A$ BREAKING

There is a theoretical complication with the construction given above. The trick was to remove all contributions in the vicinity of zero virtuality and thereby remove all effects of chiral symmetry breaking which can contribute in the $m \to 0$ limit, *i.e.*, all effects of spontaneous chiral symmetry breaking. As advertised in previous sections, the removal of these modes eliminates the effects of chiral symmetry breaking and only the effects of chiral symmetry breaking. However, these modes do more than simply contribute to spontaneous $S(2)_L \times SU(2)_R$ chiral symmetry breaking—they also are responsible of $U(1)_A$ breaking effects. Recall that the $U(1)$ chiral symmetry is broken both spontaneously and anomalously.

Ideally one would like an algorithm to turn off the dynamical effects of $S(2)_L \times SU(2)_R$ chiral symmetry breaking and $U(1)$ chiral symmetry *separately* in order to isolate the individual effects. At first blush it might seem that it should be straightforward to do this since the two chiral symmetries are different. As will be discussed in this section, this is not necessarily possible.

The reason that one cannot easily turn off the effects separately is quite simple: observables associated with $U(1)$ axial symmetry breaking may also be associated with $S(2)_L \times SU(2)_R$ chiral symmetry breaking. Consider as a simple example the chiral condensate: $\langle \overline{q}q\rangle$. Clearly a nonzero chiral condensate violates $S(2)_L \times SU(2)_R$ symmetry since under an axial rotation in the a direction $\langle \overline{q}q\rangle \to \langle \overline{q}i\gamma_5\tau^a q\rangle$; but it also violates the $U(1)$ chiral symmetry since under $U(1)_A$, $\langle \overline{q}q\rangle \to \langle \overline{q}i\gamma_5 q\rangle$. Thus *any* method which decouples the effect of the chiral condensate in order to remove $S(2)_L \times SU(2)_R$ breaking effects must do violence to the $U(1)$ chiral symmetry. This is sufficient to show that one cannot generally separately remove the effect of $S(2)_L \times SU(2)_R$ independently from $U(1)_A$.

In fact, this connection is rather widespread. An important class of examples are the correlators of baryon currents. It turns out that when one constructs a chiral-parity multiplet of baryon currents, states of opposite parity are connected by both $SU(2)_A$ and $U(1)_A$ operators. This in turn implies that when effective

$S(2)_L \times SU(2)_R$ restoration occurs for these observables, one simultaneously has $U(1)_A$ restoration. Thus as a very practical matter it is meaningless to ask what happens if the effects of $SU(2)_L \times SU(2)_R$ spontaneous symmetry breaking are dcoupled while effects of $U(1)_A$ breaking are not. Of course, consideration of the $U(1)_A$ current is complicated by the fact that the current is broken both anomalously and spontaneously. From the preceding argument it is clear that for baryon multiplets containing no isoscalars the effects of *both* the spontaneous and anomolous $U(1)_A$ breaking must become small for effective $SU(2)_L \times SU(2)_R$ restoration to occur. For case such as these, it is clearly pointless to try to separate the two effects.

Of course, more generally there are cases where the two effects are distinct. The strategy adopted here is conservative. Namely, the effects of both anomalous and spontaneous $U(1)_A$ breaking and of the spontaneous breaking $SU(2)_L \times SU(2)_R$ are suppressed. Clearly if one sees effective restoration with this prescription, then restoration of $SU(2)_L \times SU(2)_R$ has occurred; the converse, however, need not be true. Fortunately, there is phenomenlogical evidence for the onset of effective $U(1)_A$ restoration along with $SU(2)_L \times SU(2)_R$ in the meson spectrum[3, 4, 5].

VI. PRACTICAL IMPLEMENTATION VIA LATTICE QCD

The preceding argument makes clear that the extent to which effective chiral restoration occurs can be quantified in QCD spectral functions. Thus, the issue of principal is settled. Effective chiral restoration is a sensible notion in QCD. What is not clear is whether it occurs in a regime where discernible resonances exist. It also remains to be seen whether or not the idea can be tested in practice via numerical simulations of lattice QCD. Ideally a lattice version of the algorithm could be implemented straightforwardly. One would take a large volume, then calculate enough states to map out the spectral function and then repeat the calculation with the spaded currents. Both m and λ_0 should be varied to make sure that the appropriate limits are being simulated.

In practice, it will be a considerable time before it is possible to implement this in a completely unambiguous way. The principle problem is *not* the calculation of the correlators with the spaded currents. In practice this amounts to removing a set of modes with low virtuality from the calculation of quark propagators. Calculations of this sort are doable. Indeed, studies of this sort have been done recently by Degrand [13] precisely to study the role of the modes responsible for chiral symmetry breaking on the correlators. In fact, these correlators reveal that the modes of small virtuality contribute principally to the long-distance part of the correlators. This is consistent with the idea of effective chiral restoration for high mass states.

However, for a direct test of the idea of effective chiral restoration one needs more than the correlators; one needs the spectral functions. The method to extract spectral information from correlators is clear in principle. One works with a finite volume which discretizes the spectrum and extracts as many discrete states as is numerically possible. One standard way to do this is via correlators of multiple sources with the same quantum numbers [14, 15]. Since the discrete states become increasingly dense as the box size increases, one would ideally take a large size and thereby carefully fill out the spectral function. By varying the box size one can sweep through the spectrum.

Unfortunately, the numerical noise grows exponentially as one one goes to higher states. Thus, to go up to a fixed moderately high mass region of interest, the cost of going to a large box is that one needs exponentially accurate correlators. So, in practice one must settle for moderately small box sizes and moderately low-lying excitations. It is not clear that present-day lattice simulations are sufficient to extract resonances reliably. The present state of the art for extracting excited states from lattice QCD can be seen in ref. [16].

The restriction to moderately small volume complicates the analysis in two ways. The first difficulty with small volumes is that there is not enough information to scan through the spectral functions. One only has information about a few discrete values of energy. In principle one can vary the volume (while keeping it moderately small) and use the motion of the discrete levels as a function of volume to sweep through the spectrum. In practice this is likely to be prohibitively expensive for some time to come. More likely, initial calculations to extract resonant states will be done with only a couple of different volumes. In general this not sufficient to reproduce the spectral function even approximately. However, if a state is found with a mass which is largely insensitive to the choice of volume and whose coupling to the external currents is also insensitive to the volume, it is probably safe to associate the state with a relatively narrow resonance. To the extent such an algorithm can be used to reliably pick out resonant states, it can be used to verify effective chiral restoration. One can repeat the same calculation using the spaded current. The extent to which the mass *and* the coupling to the external current remain unaltered by this is a direct measurement of effective chiral restoration. This suggests at least a crude test of effective chiral restoration on the lattice might only be slightly more difficult than the task of extracting resonant states in the first place.

Unfortunately, there is an additional complication caused by the need to use relatively small volumes. This is a restriction on the appropriate value of λ_0. To work in a sensible way near the appropriate limit, λ_0 must be very small compared to explicit chiral symmetry breaking effects while being very large compared to effects associated with the finite size:

$$V m_\pi^2 f_\pi^2 \gg V \lambda_0^4 \gg 1 \ . \tag{22}$$

For small V, these inequalities become impossible to maintain unless the explicit chiral symmetry breaking terms become large since a relatively large pion mass is needed. However, as the explicit chiral symmetry breaking terms become large, one destroys the very symmetry of interest. Of course, in lattice calculations there is *always* as an interplay between the infinite volume and chiral limits: as one takes quark masses to be small one must take volumes to be large or the pionic tails of hadronic wave functions will not fit on the lattice. For typical lattice applications the relevant condition is $V m_\pi^2 f_\pi^2 \gg 1$. However, inequality (22) is stronger: one needs to fit a new scale between $V m_\pi^2 f_\pi^2$ and unity.

¿From these considerations it seems apparent that practical tests of the idea of effective chiral restoration will initially be constrained to unrealistically large quark masses. Potentially this may undermine the point of the exercise—to test the chiral properties of the states. However, it should be noted that the idea underlying effective chiral restoration—that the high-lying states become insensitive to the dynamics of dynamical symmetry breaking due to their scales—may well also apply to explicit symmetry breaking. Thus, if a test of the idea with unrealistically large quark masses indicates a considerable degree of effective chiral restoration, it is highly plausible that the real case will have a higher level of effective restoration. Thus, while the absence of convincing evidence of effective chiral restoration for lattice calculations of some moderately low-lying hadrons will not rule out the possibility, seeing the effect will provide strong evidence for the idea.

In summary, this paper has shown that the notion of effective chiral restoration is definable in QCD: the condition is that spectral functions for spaded currents are similar to those of the usual currents. To the extent that lattice calculations can be used reliably to extract resonances via the calculation of the idea ought to be testable.

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